A Novel Numerical Approach for Electromagnetic Scattering: The Cell Method

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1 Introduction

The scattering and radiation problems, with arbitrary geometries and non homogeneous media, have always been difficult to solve. The main difficulties lie mainly on using:
1. Reliable radiation boundary conditions,
2. A reliable modeling of curved shapes,
3. An efficient modeling of arbitrary material media.
The Method of Moments (MoM) has been one of the first methods overcoming the difficulties of the item 1, including proper radiation boundary conditions, and the item 2 choosing suitable interpolating functions. The disadvantages of the MoM are a lack of versatility to meet the item 3 and a heavy computational effort to solve the resulting linear system composed of full matrices. The development of reliable absorbing boundary conditions for the Finite Element Method (FEM) and the Finite Difference in Time Domain Method (FDTD) have attracted more preference for these methods in order to solve the problems of scattering and radiation.
The FEM is extremely versatile because with the PML it meets the item 1, it works on unstructured grids (item 2) and takes into account easily non homogeneous media (item 3) . Moreover, it exploits well the local character of the differential equations that leads to sparse matrices linear systems. Even if the size of these linear systems are larger than those ones from MoM, thanks to the sparsity of the FEM matrices the time to solve the first tends to be smaller. The main disadvantage of the FEM, the impossibility to have explicit algorithms in time domain, is the big advantage of the FDTD.
The FDTD meets the items 1 and 3 but it has some difficulties in modeling curved shapes. In order to have all the advantages of the FEM and to work in the time domain with explicit algorithms, another category of methods have been developed. These methods such as the Discrete Surface Integration (DSI) [1], Finite Integration Theory (FIT) [2], Cell Method (CM) [3][4], which we call briefly Finite Volume like Methods (FV), are mainly characterized by:

- Use of two staggered grids rather than only one like in MoM and FEM. These two grids, that we call primal and dual grid, can be cartesian orthogonal grids with the parallelepiped as primal basic cell, Delaunay-Voronoi grids or Delaunay-Barycentric grids [4] with the tetrahedron as primal basic cell, or other more general.
- Separate discretization of the field equations and the constitutive relations, rather than a discretization of the final integral equations (MoM) or final differential equations (FEM, FDTD) that describes completely the phenomenon we want to study.

It is important to notice that, while for the field equations the characterization is the same, but expressed in terms of local field variables **E**,**D**,**H**,**B**,**J** (DSI, FIT) or global (integral) field variables V,Ψ,F,Φ,I (CM), the situation is not the same for the constitutive relations, since they are strictly dependent on the kind of basic cell (tethraedron, parallelepiped or other) and staggered grids used. In this article , in order to build the constitutive relations for the CM, a new kind of interpolation scheme, the microcell interpolation scheme [4], is proposed . This interpolation scheme can work with many kind of basic cells (tethraedrons, parallelepipeds or other) always using the same basic interpolating function (uniform field). Then combining the field equations and the constitutive relations a finite formulation of the dielectric scattering problem will be obtained. Finally some comparisons of the results of this approach, here in the frequency domain, with the results of MoM, FEM and theoretical results (Mie's problem) will be provided.

2 Cell Method

The CM is a numerical methodology, born from a deep analysis of the formal structures common to many physical theories [3], that permits to formulate the field equations directly in a topological form. Starting from a space discretization by two staggered grids (primal and dual grid) the main idea of CM is the use of the physical global variables (electric voltage V, electric flux Ψ, magnetic voltage F, magnetic flux Φ and electric current I) through their coherent association to the oriented geometrical elements (electric voltage V-primal line L, magnetic flux Φ-primal surface S,...). Thus the electrodynamics laws, that are links between physical variables, can be divided into two categories, according into the type of variables that are linked. There are the *Topological equations* (field equations), that link physical variables associated to geometrical elements belonging to the same grid (the primal or the dual one), and *Constitutive relations*, that link physical variables associated to geometrical elements belonging to different grids (i.e. the electric constitutive relation links V on primal lines L with Ψ on dual surfaces $\tilde{S}$).

3 Topological equations

Let us denote with $[G]$, $[C]$, $[D]$ e $[\tilde{G}] = [D]^T$, $[\tilde{C}] = [C]^T$, $[\tilde{D}] = -[G]^T$, the incidence matrices related to the to geometrical elements belonging to the primal and dual grids respectively, which are the discrete counterparts of the operators $grad$, $curl$, div. With these discrete operators the topological equations, in the frequency domain, can be expressed as follows:

Faraday-Neumann's law: $[C]\{V\} = -j\omega\{\Phi\}$, on the primal grid
Maxwell-Ampère's law: $[\tilde{C}]\{F\} = j\omega\{\Psi\} + \{I\}$, on the dual grid
where $\{V\}$, $\{\Phi\}$, $\{F\}$, $\{\Psi\}$, $\{I\}$ are scalar arrays.

4 Constitutive relations

The constitutive relations can be expressed as follows:
Electric constitutive relation:$\{\Psi\} = [M_\varepsilon]\{V\}$
Magnetic constitutive relation:$\{F\} = [M_\mu^{-1}]\{\Phi\}$
where $[M_\varepsilon]$ is the electric constitutive matrix and $[M_\mu^{-1}]$ is the magnetic constitutive matrix. In order to build these matrices the microcell interpolation scheme [4] will be used. The basic geometrical structure of this idea is the *microcell*, a hexahedrical cell defined by the

intersection of a primal cell v and a dual cell $\tilde{v}$. The advantage of this interpolation scheme is the possibility of using the same interpolating functions (uniform field inside each microcell) even if the shape of the primal cells changes, since the microcells are always hexaedra; moreover the resulting constitutive matrices $[M_\varepsilon]$ and $[M_\mu^{-1}]$ are sparse. The main disadvantage of the present interpolation scheme is that in general the matrices $[M_\varepsilon]$ and $[M_\mu^{-1}]$ are not symmetric (or not hermitian). Despite the lack of symmetry does not affect the accuracy of the results, it is clear that in order to memorize a symmetric matrix we need half memory and the resolution of symmetric linear systems is faster in general.

5 Formulation

Let us consider a 3D space, already discretized by two staggered grids, with a dielectric body inside (for simplicity we assume $\mu = \mu_0$). Given the array of the voltages V_i of the incident field, the voltages of the scattered field satisfy the equation:

$$([C^T M_{\mu_0}{}^{-1} C] - \omega^2 [M_\varepsilon])\{V_s\} = \omega^2([M_\varepsilon] - [M_{\varepsilon_0}])\{V_i\} \tag{1}$$

where an anisotropic PML as absorbing boundary condition is used. Lastly the scattered far field is obtained in the conventional way by integrating the surface equivalent induced magnetic $\mathbf{M_s}$ and electric $\mathbf{J_s}$ currents calculated from V_s through an interpolation on the primal cells.

6 Numerical results

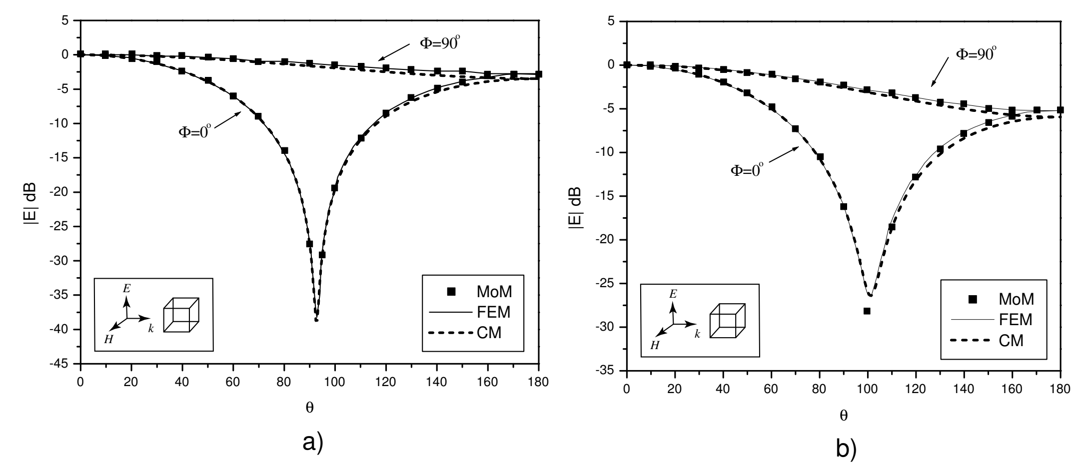

Figure 1: Scattered far field for a dielectric cube with edge $l = 0.2\lambda$ a)$\varepsilon_r = 4$ b)$\varepsilon_r = 9$

In the first and second test a plane wave impinges on a dielectric cube with edge $l = 0.2\lambda$ and respectively $\varepsilon_r = 4$ and $\varepsilon_r = 9$. The scattered far field, calculated by MoM (480 unknowns), FEM-edge elements (4974 unknowns) and CM (3088 unknowns over half domain) is displayed in Fig.1a and in Fig.1b. In the third test the plane wave impinges on a dielectric sphere with $\varepsilon_r = 9$ and radius $R = 0.408/(2\pi\lambda)$ (Mie's problem). Given the Radar Cross Section (RCS) the quantity $\frac{RCS}{\pi R^2}$ calculated by FEM-edge elements (17334 unknowns) and CM (4774 unknowns over half domain) is displayed in Fig.2a. In Fig.2b there is a comparison of the relative errors of the FEM solution and the CM solution compared with the exact solution of the Mie's problem.

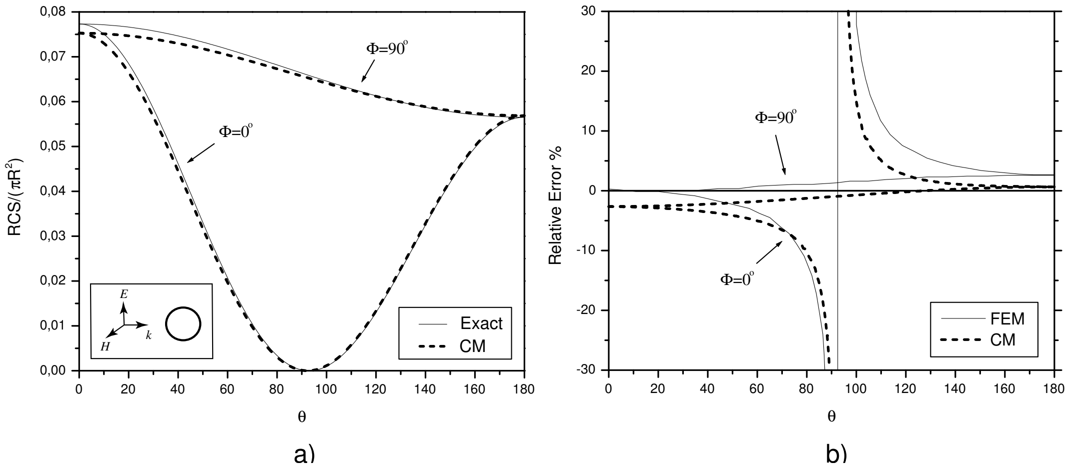

Figure 2: Scattering for a dielectric sphere with $\varepsilon_r = 4$ and radius $R = 0.0649\lambda$ a)Normalized Radar Cross Section b)Relative error

7 Conclusions

In this paper a new interpolation scheme is proposed in the Cell Method, a Finite Volume like Method, in order to solve scattering problems in the frequency domain. The numerical results, showing an excellent agreement both with the results of MoM, FEM and with the theoretical results, confirm the validity and the accuracy of the present approach. Despite the present interpolation scheme leads to non symmetric constitutive matrices, in time domain explicit algorithms can be obtained where such non-symmetric matrices seem to be transformed, for tethraedrical cells, in symmetric semi-definite positive matrices without any approximation. This subject, however, needs further work and investigations.

References

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