Plates: normal load

differential formulation

configuration variables: geometrical variables
space primal complex

source variables: statical variables
space dual complex

Lagrange-Germain

$$\nabla^4 w\,(x,y) = \frac{q\,(x,y)}{D}$$

$1\mathbf{P}$: w

$$\chi_x = -\partial_{xx} w$$
$$\chi_y = -\partial_{yy} w$$
$$\chi_{xy} = -\partial_{xy} w$$

$2\mathbf{L}$: $\chi_x, \chi_y, \chi_{xy}$

$$-\partial_y \chi_x - \partial_x \chi_{xy} = \eta_x$$
$$-\partial_y \chi_{xy} - \partial_x \chi_y = \eta_y$$

$1\mathbf{S}$: η_x, η_y

$\eta_x = 0$
$\eta_y = 0$

displacement method

$$\left\{\begin{array}{c} M_x \\ M_y \\ M_{xy} \end{array}\right\} = \left[\begin{array}{ccc} D & \nu D & 0 \\ \nu D & D & 0 \\ 0 & 0 & (1-\nu)\,D \end{array}\right] \left\{\begin{array}{c} \chi_x \\ \chi_y \\ \chi_{xy} \end{array}\right\}$$

force method

$1\tilde{\mathbf{V}}$: q

equilibrium

$$-\partial_{xx} M_x - \partial_{yy} M_y - 2\partial_{xy} M_{xy} = q$$

$2\tilde{\mathbf{S}}$: M_x, M_y, M_{xy}

$$M_x = \partial_y \cdot i_x$$
$$M_y = -\partial_x \cdot i_y$$
$$M_{xy} = \frac{1}{2}\left(\partial_y \cdot i_y = -\partial_x \cdot i_x\right)$$

$1\tilde{\mathbf{L}}$: $\cdot i_x, \cdot i_y$

$$D = \frac{Eh^3}{12\,(1-\nu^2)}$$

w	*vertical deflection*
$\chi_x, \chi_y, \chi_{xy}$	*flectional and twisting curvatures*
η_x, η_y	*incompatibiliy functions*
q	*normal load for unit surface*
M_x, M_y, M_{xy}	*bending and twisting moments*
$\cdot i_x, \cdot i_y$	*stress functions*
D	*bending rigidity*

Ref: Fung Y.C., Foundations of Solids Mechanics, Prentice Hall, Inc., 1965, p.456