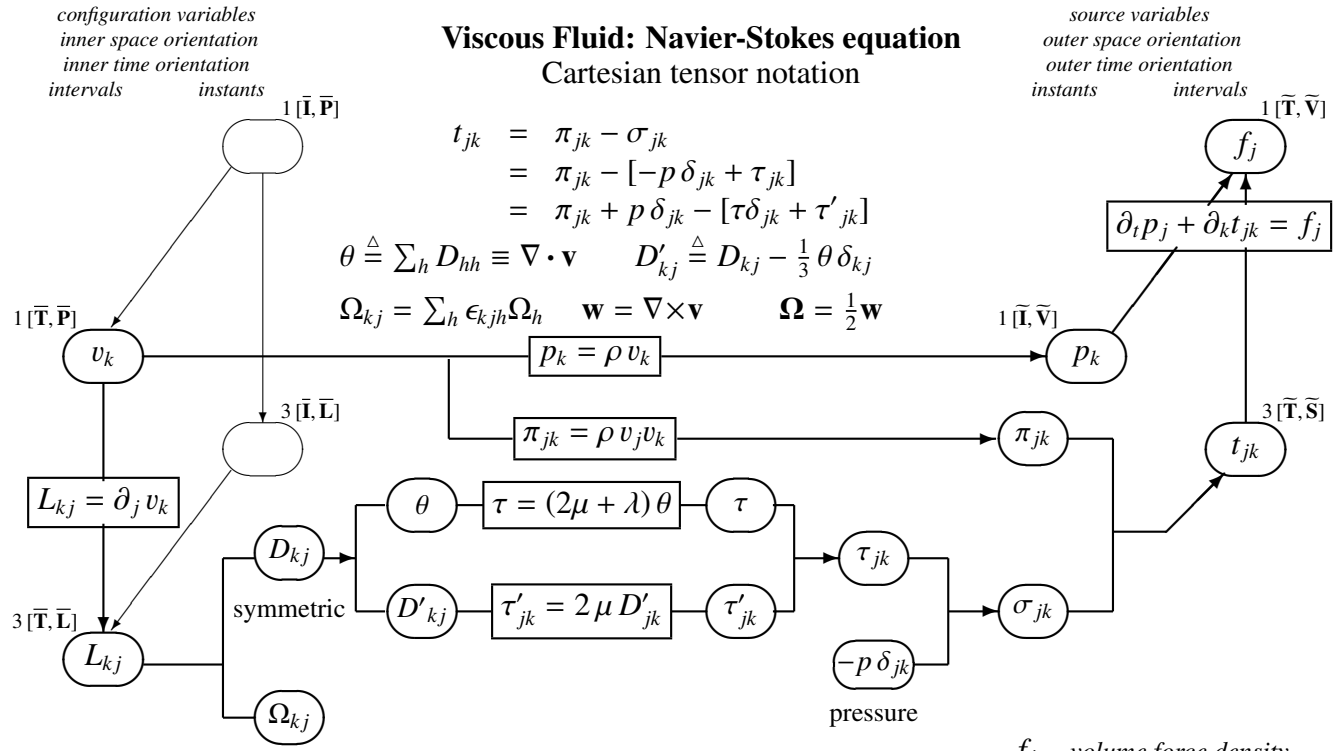
Viscous Fluid: Navier-Stokes equation

Cartesian tensor notation

$$
\begin{aligned}
t_{jk} &= \pi_{jk} - \sigma_{jk} \\
&= \pi_{jk} - [-p\,\delta_{jk} + \tau_{jk}] \\
&= \pi_{jk} + p\,\delta_{jk} - [\tau\delta_{jk} + \tau'_{jk}]
\end{aligned}
$$

$$\theta \triangleq \sum_h D_{hh} \equiv \nabla \cdot \mathbf{v} \qquad D'_{kj} \triangleq D_{kj} - \tfrac{1}{3}\,\theta\,\delta_{kj}$$

$$\Omega_{kj} = \sum_h \epsilon_{kjh}\Omega_h \qquad \mathbf{w} = \nabla \times \mathbf{v} \qquad \mathbf{\Omega} = \tfrac{1}{2}\mathbf{w}$$



v_k	*velocity*	D_{kj}	*strain rate tensor*	σ_{kj}	*symmetric stress tensor*
L_{kj}	*velocity gradient*	θ	*volume dilatation rate*	τ'_{kj}	*viscous stress deviator*
Ω_{kj}	*spin tensor*	D'_{kj}	*strain rate deviator*	τ	*trace of* τ_{kj}

f_j	*volume force density*
p_j	*momentum density*
t_{kj}	*total stress tensor*
τ_{kj}	*viscous stress tensor*
π_{kj}	*dynamical stress tensor*

Ref: Hunter, p.115