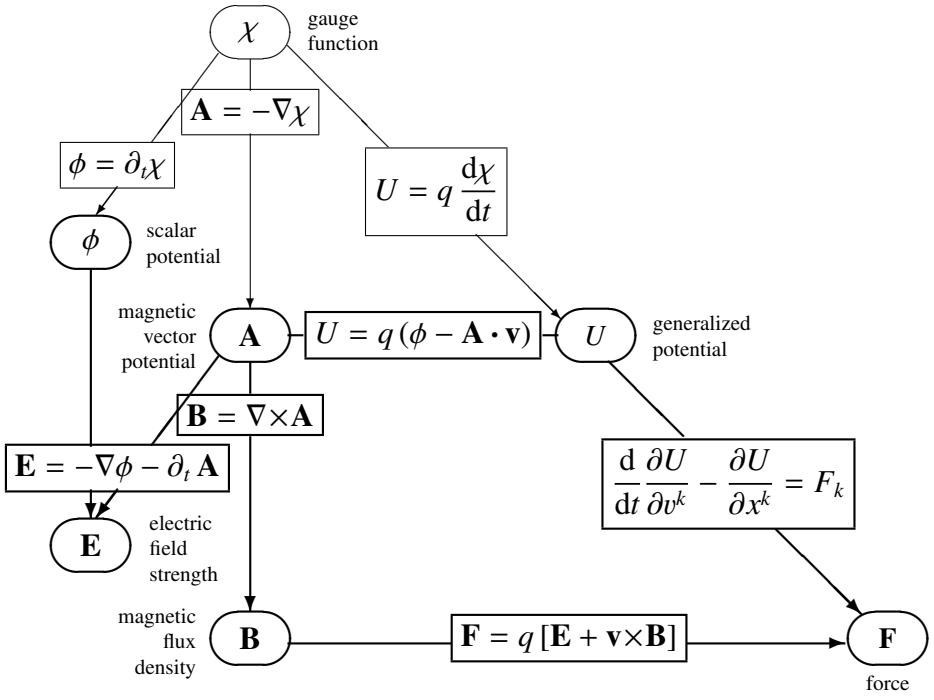
Particle motion in the electromagnetic field



$$U(t,\mathbf{x},\mathbf{v}) = q\left[\phi(t,\mathbf{x}) - A_h(t,\mathbf{x})\,v^h(t)\right]$$

$$\begin{aligned}
F_k &= \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial U}{\partial v^k} - \frac{\partial U}{\partial x^k} \\
&= q\left[-\frac{\mathrm{d}}{\mathrm{d}t}A_k - \left(\partial_k\phi - \partial_k A_h\,v^h\right)\right] \\
&= q\left[-\partial_t A_k - \partial_h A_k\,v^h - \partial_k\phi + \partial_k A_h\,v^h\right] \\
&= q\left[E_k + (\partial_k A_h - \partial_h A_k)\,v^h\right] \\
&= q\left[E_k + B_{kh}\,v^h\right] \\
\mathbf{F} &= q\,[\mathbf{E} + \mathbf{v}\times\mathbf{B}]
\end{aligned}$$

Ref.: Goldstein H., Classical Mechanics, Addison Wesley, 1957, p.21